Mathematical Modeling of Wear Characteristics of 6061 AI-Alloy-SiCp Composite Using Response Surface Methodology

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In the light of attractive wear characteristics as well as high strength to weight ratio, extensive research on Al-based Metal Matrix Composite (MMC) have been carried out globally in the last two decades. However, very limited research has been pursued on tribological behavior of Al-based MMC under combined action of rolling and sliding. This study investigates the wear behavior of 6061 Al-alloy/SiC with 10 vol.% SiCp against hardened and tempered AISI 4340 steel under combined rolling-sliding conditions. 2³ factorial design of experiments have been carried out to see the effect of few parameters, i.e., contact stress, speed and duration with respect to wear. The interaction effect has also been studied by 3D graphical contours. A mathematical model is developed using regression analysis technique for prediction of wear behavior of the MMC and adequacy of the model has been validated using analysis of variance (ANOVA) techniques. Finally, the optimization of parameter has also been done using Design Expert software. The results have shown that Response Surface Methodology (RSM) is an effective tool for prediction of wear behavior under combined sliding and rolling action. It is also found that the wear of MMC is much lower than hardened; tempered AISI 4340 steel and rolling speed has the maximum influence in wear of both materials under investigation.

Keywords analysis of variance, metal matrix composite, rollingsliding conditions

1. Introduction

The increased demand of light weight materials with high strength to weight ratio in the aerospace and automotive industries has led to the development and use of Al-alloy-based composites (mainly Al-alloy/SiCp composites). The Metal Matrix Composites (MMCs) are slowly replacing the general light metal alloy such as aluminum alloy in different industrial application where strength, low weight and energy savings are the most important criteria.

The combination of various properties like electrical, mechanical, and even chemical can be achieved by use of different types of reinforcements, i.e., continuous, discontinuous, short, whiskers, etc., with the MMCs (Ref 1, 2). SiC

particles can readily be processed using different machining techniques like extrusion, pressing, rolling, etc., and very much compatible with aluminum matrix. Dry wear, friction properties, tool wear, and surface roughness of Al₂O₃ reinforced Al alloy MMC have been studied in the article of Akbulut et al. (Ref 3) and Sahin et al. (Ref 4), respectively. But a limited number of investigations have been done to study the abrasive behavior of ceramic particles reinforced Al-alloy composite. One such investigation has been done in the article of Prasad (Ref 5) where the combined effect of high load and coarse abrasive size has been studied using Zn-Al alloy/SiCp composites. The two-body abrasive wear behavior of a cast Al-alloy and 10 wt.% Al₂O₃ particle composite was studied by Mondal et al. (Ref 6) at different loads (1 to 7 N) and abrasive sizes (30 to 80 µm). The wear behavior was also estimated using statistical analysis at different operating conditions. The different metallurgical factors and their interaction will greatly affect the abrasive wear of Al composites (Ref 7). So for analyzing the different types of wear behavior of Al/SiC composites, few researchers (Ref 8, 9) used design of experiments in their study. With more potential friction and wear applications envisioned in the future, it is thus useful to have a greater pool of tribological data for these composites. Many experiments on MMCs in dry sliding condition have been performed to explore the wear characteristics of these materials (Ref 10-13). Some studies on drilling of hybrid metal matrix composites based on Taguchi Technique have been carried out by Basavarajappa et al. (Ref 14). However, a survey of the technical literature reveals that the tribological data available for this group of Al-alloy/SiCp is mainly concentrated on sliding wear. As a result, in this study, 6061 Aluminum alloy/SiCp has been considered to study the wear behavior

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under rolling-sliding conditions using lubricant. This study concentrates on the wear behavior of particle reinforced metal matrix composite (6061Al-alloy/SiCp) against hardened and tempered AISI 4340 Steel under combined rolling and sliding actions in lubricated conditions. The combined effects of the process parameters such as, contact stress, speed, and duration of running and their interactions are also investigated. A mathematical model is developed using regression analysis technique for prediction of wear behavior of the MMC. Adequacy of the developed model has been validated using analysis of variance (ANOVA) techniques. Finally, the optimization of the parameter has also been done.

2. Experimental Details

2.1 Materials

The MMC, using 6061 Al-alloy (Al-Si-Mg alloy) as the base metal and 10% volume fraction (V_f) of SiCp particles as reinforcement material was prepared by Vortex Method. Though there are several fabrication techniques available to manufacture MMC materials has been described by Naher et al. (Ref 15), depending on the choice of matrix and reinforcement material, the fabrication techniques can vary considerably. Fabrication methods can be divided into three types. These processes have been discussed in the article of Skibo et al. (Ref 16), i.e., solid phase processes, liquid phase process, and semisolid fabrication process. Solid state processes are generally used to obtain the best mechanical properties in MMCs, particularly in discontinuous MMCs. This is because segregation effects and intermetallic phase formations are less for these processes, when compared with liquid state processes. The advantages of using Vortex method (also known as stir casting) lies in its simplicity, flexibility, highly economical when compared to other processes and also to its applicability to large-scale production. However, it requires optimum process control for mass production. The chemical composition of the base metal and the details of SiCp are given in Table 1.

2.2 Preparation of MMC

The particles used for MMC in this study were a type of α -SiC particles with average diameter of 23 µm. The volume fraction of SiC particles added to the melt was restricted to 10%. The reason for using α -SiCp is that it has a high hardness, a low coefficient of thermal expansion and a good wetting property. The SiC particles were mixed and dispersed in the molten 6061 Al-alloy using the Vortex Method. A schematic view of the vortex apparatus used in this process has been presented in Fig. 1. The 6061 Al-alloy was melted in a crucible and then stirred at high speed to create a vortex by stainless steel agitator coated with molybdenum using the plasma spray method. The SiC particles were then gradually added and stirred in. During this stage 1-2% calcium was added to the melt as wetting agent. The effect of calcium is that it accumulates in high concentrations in the vicinity of the surface of SiC particles. It reduces the surface tension of aluminium as well as increases the wetting properties of aluminium and SiC. In this way, mixing and dispersion time also reduces a large extent. It was possible to disperse the particles evenly after 60 min of stirring. The whole process of melting and mixing was carried out under an inert atmosphere

Table 1Chemical compositions of selected materialalong with volume fraction of SiCp and particle sizeof SiCp

Si	Mg	Cu	Fe	Cr	Al	Vol.% SiCp	SiC particle size, µm
0.6	1.0	0.28	0.6	0.2	97.9	10	23
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Reinforce particle fraction in vol.%

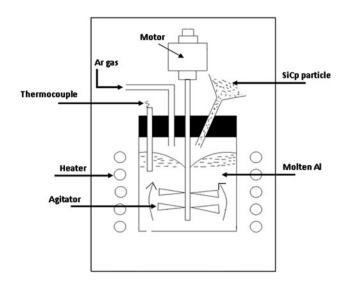


Fig. 1 Schematic diagram of vortex method for developing MMC (6061Al-alloy/SiCp)

Table 2Variables used in vortex method for preparationof MMC

Base metal	Temperature of melt, °C	Rotation speed, rpm	Adding rate, g/min	Pre-heat temp. of particles, °C
6061Al-alloy	750	800-1000	50	500

of argon gas. The conditions used for the vortex-method are presented in Table 2.

The important points in these conditions are the temperature of the molten Al-alloy and the speed of the agitator. If the temperature of the molten Al-alloy is too low, it will not be possible to create a vortex and if it is too high, there is the possibility that the SiCp and aluminum will react with each other; the temperature of the molten aluminum was therefore set at 50-100 °C above the melting point. If the stirring speed is too low, it will not be possible to create a vortex and if it is too fast, the added SiC particles will be liable to be scattered, so the optimum speed was maintained at 800 to 1000 rpm throughout the experiments. After dispersing the SiC particles in the molten Al-alloy, the resulting MMC melt was poured into a mold of 60 mm inner diameter where it solidified as a billet. The as-cast billets of 6061Al-alloy/SiCp (particle size = 23 μ m and $V_{\rm f} = 10\%$) of 60 mm diameter and with a hardness value of 46 HB were turned down to 45 mm diameter by machining and then were annealed as shown in at 415 $\,^{\rm o}{\rm C}$ for 3-4 h to achieve a hardness of 35 to 37 HB.

2.3 Experimental Set-Up

AMSLER machine was used to study the wear characteristics of composite due to combined rolling and sliding actions. The annular disks of 40 mm diameter and 10 mm thick as shown in Fig. 2 were made out of Al-alloy composite as well as AISI 4340 Steel in annealed and heat-treated conditions, respectively. These disks were mounted on the end of shaft so

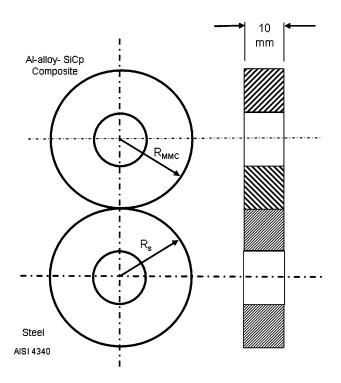


Fig. 2 Test sample for AMSLER testing machine

 Table 3
 2³ design matrix for experiments

Sl no		Contact stress, N/mm ²	Duration, h	Speed, RPM
01	Lower level (-1)	68.01	0.333	200
02	Middle level (0)	82.15	0.417	300
03	Higher level (-1)	96.29	0.501	400

that they could make contact with each other tangentially. The radial force with which they were pressed together could be regulated between 0 and 2 kN by the tension of a spring. The load applied at any time was measured directly from the attached scale. The direction of rotation and the speed of the rotation of the two disks were different, so that when using the specimen of the same diameter, there was a slip of 10%. This confirmed that 90% rolling and 10% sliding actions were applied over the test specimens. The wear of the test disks caused by friction was determined by the difference of weights before and after each test. Due to high contact stress developed at the contact point, Grade SAE 20 W/50 was used to lubricate the rolling and sliding line.

2.4 Factorial Design of Experiments

A factorial design of experiments of the type P^n was used in this study (Ref 17) where "*n*" corresponds to the number of factors and "*P*" stands for the number of levels. In this design, n = 3 (i.e., contact stress, duration, speed) and P = 2 (i.e., upper and lower levels of each variable). Thus minimum number of trial experiments to be conducted for each material is $2^3 = 8$. If total wear is represented by *Y*, the linear regression equation for these experiments can be written as;

$$Y = b_0 + b_1A + b_2B + b_3C + b_{12}AB + b_{23}BC + b_{13}AC + b_{123}ABC$$
(Eq 1)

where b_0 is the response variable (i.e., total wear) at the base level (i.e., 147.15 N load, 25 min duration and 300 rpm speed), b_1 , b_2 , b_3 are coefficients associated with each variable A (contact stress), B (time duration) and C (speed) respectively, b_{12} , b_{23} , b_{13} , and b_{123} are interaction coefficients between AB, BC, CA, ABC respectively, within the selected levels of each of the variables. The positive value of Y from Eq 1 denotes weight loss while its negative value indicates weight gain and A, B, C is the coded values of load, time duration, and speed, respectively. The coded value for a particular variable is defined as follows:

Coded value =

Base value – Selected value

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Base value - Value corresponding to the lower to upper level
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The parameters of Eq 1 have been estimated by the method of least squares using a MATLAB computer package. Upper, lower and base levels of the variables along with their coded values are tabulated in Table 3. The factorial design of experiments and the values of response variable corresponding to each sets of trial are represented in Table 4.

Table 4	The value of individual variables with their coded value	ue and wear response in each trial (as per design matrix)

Trial run	Letter symbol	Contact stress, MPa, x ₁	Time duration, h, x ₂	Rolling speed, RPM, <i>x</i> ₃	MMC wear, gm, $y \times 10^{-3}$	AISI 4340 steel wear, gm, y*
1	1	-1	-1	-1	9.7000	0.13707000
2	а	+1	-1	-1	13.4900	0.14093000
3	b	-1	+1	-1	11.7900	0.16701700
4	ab	+1	+1	-1	19.3350	0.25081000
5	с	-1	-1	+1	21.1850	0.30344500
6	ac	+1	-1	+1	20.1150	0.29400000
7	bc	-1	+1	+1	24.6750	0.41730500
8	abc	+1	+1	+1	20.4425	0.43871250

3. Result and Discussion

3.1 Development of the Initial Mathematical Model

The values of coefficients of the polynomial of Eq 1 were calculated by the regression method. The Design Expert software (Version 8.0.1) used to calculate the values of the

Table 5The percentage contribution of each factoreffecting on AISI 4340 steel

Terms	Effect	Sum of Square	% Contribution
A—contact stress	0.024903875	0.001240406	1.276845838
<i>B</i> —time duration	0.099599875	0.01984027	20.42312504
C-rolling speed	0.189408875	0.071751444	73.85931215
AB	0.027696375	0.001534178	1.579248493
AC	-0.01892263	0.000716131	0.737169529
BC	0.029686375	0.001762562	1.814341139
ABC	-0.01227013	0.000301112	0.30995781

Table 6The percentage contribution of each factoreffecting on MMC wear

Term	Effect	Sum of square	% Contribution
A-contact stress	0.0015075	4.54511E-06	2.350517175
B—time duration	0.0029375	1.72578E-05	8.924924232
C-rolling speed	0.008025	0.000128801	66.60991347
AB	0.0001475	4.35125E-08	0.022502607
AC	-0.00416	3.46112E-05	17.89927533
BC	-0.00103	2.1218E-06	1.097294587
ABC	-0.00173	5.9858E-06	3.095572598

Table 7 ANOVA table for AISI 4340 steel wear

coefficient. The final mathematical model as determined by the analysis is given under.

AISI 4340 Steel Wear

$$= +0.27 + 0.012 * A + 0.050 * B + 0.095 * C$$

+ 0.014 * A * B - 0.009461 * A * C + 0.015 * B * C
- 0.006135 * A * B * C (Eq 2)

MMC Wear =
$$+ 0.018 + 0.0007537 * A + 0.001469 * B$$

+ $0.004012 * C + 0.00007375 * A * B$
- $0.002080 * A * C - 0.000515 * B * C$
- $0.0008650 * A * B * C$ (Eq 3)

3.2 Development of Final Mathematical Model

The contribution of each term are calculated using the same software and has been reported in Table 5 and 6 for AISI 4340 steel and MMC wear respectively. After calculating the ANOVA using forward elimination method it is seen that *B* and *C* are only significant terms for the wear of AISI 4340 steel. After applying the similar method, it has been concluded that *A*, *B*, *C* and *AC* are only significant terms for the wear of MMC (Table 7, 8). Hence, the Eq 2 and 3 reduced to

AISI 4340 Steel Wear =
$$0.26866119 + 0.04979994 * B$$

+ $0.09470444 * C$ (Eq 4)
MMC Wear = $0.01759125 + 0.00075375 * A$
+ $0.00146875 * B + 0.0040125 * C$
- $0.00208 * A * C$ (Eq 5)

Source	Sum of squares	df	Mean square	F value	$P \text{ value} \\ P > F$	
Model	0.091591714	2	0.045795857	41.2249241	0.0008	Significant
B—time duration	0.01984027	1	0.01984027	17.8599919	0.0083	Ū.
C-rolling speed	0.071751444	1	0.071751444	64.5898564	0.0005	
Residual	0.005554389	5	0.001110878			
Cor total	0.097146104	7	R^2	0.942824372		
SD	0.03333		Adj R^2	0.919954121		
Mean	0.268661		Pred R^2	0.853630392		
CV %	12.4059		Adeq Precision	14.15996549		

Table 8 ANOVA Table for MMC wear

Source	Sum of squares	df	Mean square	F value	$P \text{ value} \\ P > F$	
Model	0.000185215	4	4.63038E-05	17.04203337	0.0211	Significant
A-contact stress	4.54511E-06	1	4.54511E-06	1.672819201	0.2865	
B—time duration	1.72578E-05	1	1.72578E-05	6.351701992	0.0862	
C-rolling speed	0.000128801	1	0.000128801	47.40503214	0.0063	
AC	3.46112E-05	1	3.46112E-05	12.73858016	0.0376	
Residual	8.15111E-06	3	2.71704E-06			
Cor total	0.000193366	7	R^2	0.957846		
SD	0.001648344		Adj R^2	0.901641		
Mean	0.01759125		Pred R^2	0.70024		
CV %	9.370248651		Adeq Precision	11.60475		

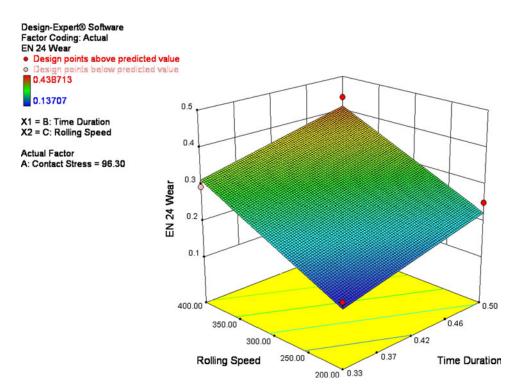


Fig. 3 Effect of wear on AISI 4340 at high contact stress

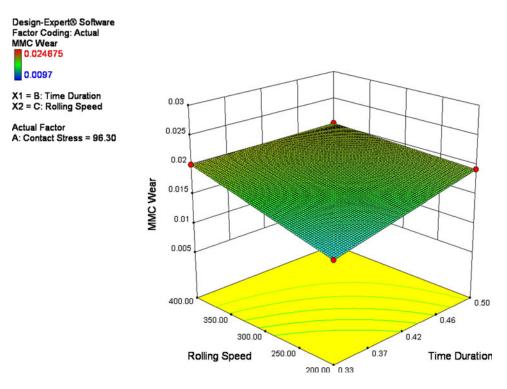


Fig. 4 Effect of wear on MMC (6061Al-alloy/SiCp) at high contact stress

3.3 Response Surface Methodology

The mathematical model described in section 3.2 can be employed to predict the wear behavior of MMC and AISI 4340 steel for the range of process parameters used in the investigation by substituting their respective values in coded form based on this model. The interaction effect of wear at high contact stress have been computed and plotted in Fig. 3 and 4 for AISI 4340 steel and MMC, respectively, and low contact stress have been computed and plotted in Fig. 5 and 6 for AISI 4340 steel and MMC, respectively. It has been seen from the figure that at

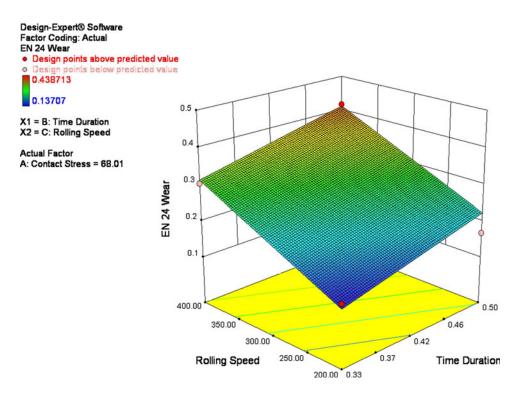


Fig. 5 Effect of wear on AISI 4340 at low contact stress

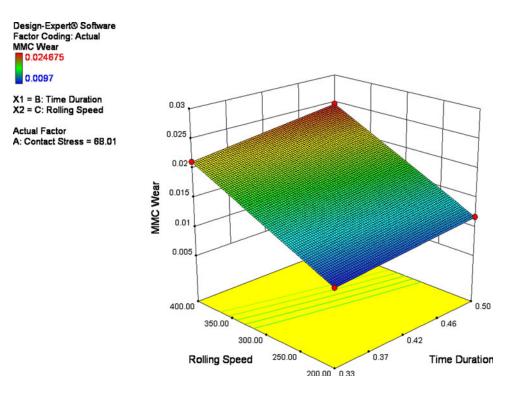


Fig. 6 Effect of wear on MMC (6061Al-alloy/SiCp) at low contact stress

low contact stress the wear rate is same for both the material but at high contact stress the wear rate on AISI 4340 steel is high when compared to MMC. In this study, desirability function optimization of the RSM has been employed for single response optimization. The optimization module searches for a combination of factor levels that simultaneously satisfy the requirements placed on each of the responses and factors in an attempt to establish the appropriate model. During the optimization process the aim was to find out the optimal values of operational parameters like speed, stress, and time to minimize the values of wear rate of AISI 4340 steel and MMC during the process. The constraints used during the optimization process

Condition	Goal	Lower limit	Upper limit
Contact Stress	Minimize	68.01	96.30
Time duration	Maximize	0.333	0.50
Rolling speed	Is in the range	200	400
AISI 4340 steel wear	Minimize	0.13707	0.438713
MMC wear	Minimize	0.0097	0.024675

Table 10Optimization result

Solution no	Contact stress, N/mm ²	Time duration, h	Rolling speed, RPM	AISI 4340 steel wear, gm	MMC wear, gm	Desirability	Remarks
1	68.01	0.50	200.00	0.223757	0.0122138	0.8775318	Selected
2	68.01	0.50	200.00	0.222712	0.0121829	0.8768234	
3	68.01	0.50	200.00	0.221558	0.0121489	0.8760055	
4	68.01	0.50	200.76	0.224474	0.0122599	0.8759846	
5	68.01	0.50	200.00	0.221176	0.0121377	0.8757252	
6	68.20	0.50	200.00	0.223756	0.0122519	0.8753775	
7	68.01	0.49	200.00	0.220021	0.0121036	0.8748602	
8	68.06	0.49	200.00	0.219304	0.0120917	0.8737801	
9	68.01	0.49	200.00	0.218564	0.0120606	0.8737093	
10	68.45	0.50	200.00	0.223756	0.0123018	0.8725515	
11	68.01	0.49	200.00	0.216798	0.0120087	0.872222	
12	68.55	0.50	200.00	0.22374	0.0123222	0.8713569	
13	68.01	0.50	203.04	0.226637	0.0123992	0.8712962	
14	68.01	0.48	200.06	0.213196	0.0119044	0.8687419	
15	68.01	0.48	200.00	0.211863	0.011863	0.8675942	
16	68.01	0.48	203.23	0.21592	0.0120892	0.8622843	
17	69.60	0.50	200.00	0.223756	0.0125325	0.8593393	
18	68.01	0.48	207.73	0.221591	0.012405	0.854678	
19	68.01	0.49	220.27	0.236393	0.0132551	0.8314096	
20	68.01	0.43	200.00	0.18399	0.0110409	0.8243827	

are summarized in Table 9. The optimal solutions are reported in Table 10 in order of decreasing desirability level.

4. Conclusion

From the experimental results and their statistical analysis, it has been observed that both the models for wear of AISI 4340 steel and MMC are found to be in significant with respect to experimental values. As the R^2 values of the model have come out 94.28 and 95.78% for AISI 4340 steel and MMC respectively, we can conclude that the prediction of future outcomes will be good. The individual effect of each variable is calculated and it is seen that Rolling Speed has maximum influence in wear of both the materials. The interactions among the variables have also a good influence on wear behavior of the materials. Interaction terms of contact stress (A) and rolling speed (C) has influence on the wear of MMC. The wear rate of 6061Al-alloy/SiCp composite in as-cast and annealed condition is much lower than that of hardened and tempered AISI 4340 steel under the same experimental conditions. The study of wear behavior of the MMC against AISI 4340 steel under the combined action of 90% rolling and 10% sliding forces is unique in nature. The optimization of the process has been done where our goal is to minimize the wear rate and after calculation we may conclude that 87.75% desirability can be achieved using the optimized conditions of parameters.

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